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The thermal neutronic soliton wave phenomenon in an infinite medium

The basic dynamics of so-called neutronic soliton waved into a subcritical infinite medium is described. After "ignition" (by means of a strong enough neutron source or a critical, power producing reactor zone) such waves propagate into an initially subcritical (poisoned) medium. Their velocities are proportional to their amplitudes – a typical non-linear soliton wave effect. A soliton reactor concept, based on this idea, may be particularly suited for small thermal light water reactors(e.g. for space heating purposes) to materialize the so-called "battery concept". Hereby, only twice, at the beginning of the fuel cycle and at the end of the useful lifetime of the whole reactor system, fuel has to be loaded and unloaded, respectively.

Das Phänomen thermischer Solitonenwellen in einem unendlichen Medium. Die Dynamik einer thermischen Solitonenwelle in ein unterkritisches (vergiftetes) unendliches Medium wird beschrieben. Nach der "Zündung" (durch eine starke Neutronenquelle oder einer kritischen, leistungserzeugenden Reaktorzone) wandern solche Wellen in die unterkritischen Zonen. Ihre Geschwindigkeit ist proportional zu ihrer Amplitude – ein typisch nicht-linearer Solitoneneffekt.
Ein solches Solitonenkonzept könnte insbesondere für kleine thermische Reaktoren (z. B. Heizreaktoren) interessant sein, um das "Batteriekonzept" zu verwirklichen. Dabei wird der Reaktor nur einmal zu Beginn beladen und am Ende der nützlichen Lebensdauer samt dem Kessel entsorgt.

1 Introduction

Hitherto, it has been shown [1–4] that a neutron fluence wave – if it is "ignited" once with a sufficiently high number of moles of source neutrons – propagates autocatalytically into fertile materials, producing power there and converting it in its "wake" partially into fissionable material and fission products. The shape of such a neutron fluence wave resembles very much a shock wave and the corresponding neutron fluence wave resembles a reciprocal catenary.

In analogy to these fast neutron systems we analyze in the following whether such a non-linear wave effect is also possible in a thermal system with $k_\infty < 1$. For this and for the sake of simplicity we will consider an infinite, graphite moderated, system with U-235 as the fissile fuel and B-10 as a burnable poison. Furthermore, the analysis will be performed in one-dimensional geometry with one space coordinate x and a monoenergetic thermal group of neutrons.

2 Establishing the wave equation

The non-linear integro-differential equation for the thermal neutron flux, Φ, controlling such a wave phenomenon in equilibrium, i. e. if all initial transients due to the "ignition geometry" have died out, is given by the self-explaining balance

$$D \cdot \Delta\Phi + \left[(\eta^{fis} - 1)\sigma_a^{fis} \cdot N_{fis} - \sigma_a^p \cdot N_{fp} - \Sigma_{a,o}^m\right]\Phi = \frac{1}{v}\Phi_t \qquad (1)$$

where

D = diffusion coefficient, in [cm]

η^{fis} = neutron yield per neutron absorbed, [-] (e.g. for U-235 = 2.08)

σ_a^{fis}, σ_a^p, σ_a^{fp} = microscopic absorption cross sections for the fissile material, the neutron poison, and the fission product pairs, respectively, in [b]

$N_{fis}(x, t)$, $N_p(x, t)$, $N_{fp}(x, t)$ = space- and time-dependent atomic particles densities of the fissionable material, the neutron poison, and the fission product pairs, respectively, in [cm^{-3}]

Φ(x, t) = space- and time-dependent thermal neutron flux, in [$cm^2 s$]$^{-1}$; (the subscript t denotes its partial time derivative)

$\Sigma_{a,0}^m$ = const. = macroscopic absorption cross-section of the moderator, in [cm^{-1}]

v = velocity of the wave phenomenon expected, in [cm/s]

In addition to Eq. (1) we use the following burn-up equations which yield the coupling between the atomic particle densities

$$\begin{aligned}
\dot{N}_{fis} &= -\sigma_a^{fis} \cdot N_{fis} \cdot \Phi \quad \text{with } N_{fis}(x,0) = N_{fis,0}\\
\dot{N}_p &= -\sigma_a^p \cdot N_p \cdot \Phi \quad \text{with } N_p(x,0) = N_{p,o}\\
\dot{N}_{fp} &= \sigma_a^{fis} \cdot N_{fis} \cdot \Phi - \sigma_a^{fp} \cdot N_{fp} \cdot \Phi \quad \text{with } N_{fp}(x,0) = 0.
\end{aligned} \qquad (2)$$

The three equations of Eq. (2) can be integrated explicitly yielding, together with their initial conditions,

$$N_{fis}(x,t) = N_{fis,0} \cdot e^{-F} \qquad (3)$$

$$N_p(x,t) = N_{p,o} \cdot e^{-\alpha F} \qquad (4)$$

$$N_{fp}(x,t) = \frac{N_{fis,0}}{\beta - 1}\left[e^{-F} - e^{-\beta F}\right] \qquad (5)$$

where

$$F = F(x,t) = \sigma_a^{fis} \int_o^t \Phi(x,\tau) \cdot d\tau \qquad (6)$$

is the (dimensionless) neutron fluence and the ratios

$$\alpha = \frac{\sigma_a^p}{\sigma_a^{fis}}, \ \beta = \frac{\sigma_a^{fp}}{\sigma_a^{fis}} \qquad (7)$$

are (dimensionless) microscopic cross section ratios.

Introducing Eqs. (3–5) into Eq. (1) we obtain an integro-differential equation for the neutron flux being

$$D \cdot \Delta\Phi + \Sigma^{fis}_{a,0} \cdot \Big[(\eta^{fis} - 1) \cdot e^{-F} - \alpha \cdot a \cdot e^{-\alpha F} - \frac{\beta}{\beta - 1} \cdot (e^{-F} - e^{-\beta F}) - b\Big] \cdot \Phi = \frac{1}{v}\Phi_t \quad (8)$$

with $\Sigma^{fis}_{a,0}$ being the initial macroscopic absorption cross section of the fissionable material in cm^{-1} and the (dimensionless) initial ratios

$$a = \frac{N_{p,0}}{N_{fis,0}} \qquad b = \frac{\Sigma^m_{a,0}}{\Sigma^{fis}_{a,0}}. \quad (9)$$

Eq. (8) can be written in the form

$$\Delta\Phi + B^2(F) \cdot \Phi = \frac{1}{vD}\Phi_t \quad (10)$$

which is not a classical Helmholtz equation since the buckling term is a transcendental function of the integrated neutron flux. This introduces a strong non-linearity into the wave equation. Only for zero-power applications, i. e. for $F = 0$, Eq. (10) reduces to the classical Helmholtz equation with $B^2 = (\eta^{fis} - 1) - a - b)\ \Sigma^{fis}_{a,0}/D$ in cm^{-2} = const. Only in the case B^2 = const. Eq. (10) can be solved by a product formulation for the space- and time-dependent functions and by separating them.

3 The soliton formulation

In order to solve Eq. (8) we try the formulation

$$F = F(x,t) = F\left[\frac{x - vt}{D}\right] = \sigma^{fis}_a \int_0^t \Phi(x,\tau) \cdot d\tau \quad (11)$$

because we conjecture that Eq. (8) possess a wave solution due to physical reasons. The "minus sign" in the (dimensionless) variable means that the direction of propagation with velocity v is to the right, i. e. into the direction of the positive x-axis.

From Eq. (11) we get successively

$$\Phi = -\frac{v}{D \cdot \sigma^{fis}_a}F' \qquad \Phi_t = \frac{v^2}{D^2 \cdot \sigma^{fis}_a}F''$$

$$\Delta\Phi = \Phi_{xx} = -\frac{v}{D^3 \cdot \sigma^{fis}_a}F''' \quad (12)$$

where the derivatives of F have to be taken with respect to the whole argument.

Introducing Eq. (12) into Eq. (8) we observe firstly that v cancels completely meaning that the resulting wave number will be independent on v, facilitating the problem significantly and justifying the formulation made. Secondly, the resulting ordinary autonomous differential equation of third order in F can be integrated once. In this integration step the integration constant has to be chosen in such a way that asymptotically for $F' = F'' = 0$ also F has to be zero. We call this the so-called first "ground-state" of the equation.

We obtain in this way and in dimensionless notation for $F \geq 0$

$$F'' + F' = \Theta(F) = D \cdot \Sigma^{fis}_{a,0} \cdot \Big[(\eta^{fis} - 1) \cdot (e^{-F} - 1) - a(e^{-\alpha F} - 1) - \frac{\beta}{\beta - 1}\Big\{\frac{1}{\beta}e^{-\beta F} - 1) - (e^{-F} - 1)\Big\} - bF\Big] \quad (13)$$

yielding an ordinary differential equation of second order being autonomous, i. e. its RHS, $\Theta(F)$, depends only on the amplitude of the function itself. All terms in (F) are negative for $F \geq 0$ except the second one because only the burn-up of the poison results in a positive reactivity effect.

As already pointed out [3, 4], Eq. (13) can be interpreted as generalized Sine-Gordon equation which is given by $u_{xx} - u_{tt} = \sin(u)$. The formulation $u = u(y) = u[(x-vt)/(1-v^2)^{1/2}]$ yields $d^2u/dy^2 = \sin(u)$ which has the solution $u(y) = 4 \arctan(e^y)$.

The RHS of Eq. (13), $\Theta(F)$, can be represented by a Fourier Series, possessing a fundamental sine-mode and higher sine-harmonics. Therefore, Eq.(13) can be considered as a Sine-Gordon equation with an additional friction term on the LHS and additionally higher harmonics on the RHS. An analytical solution of this type of equation is not known but approximations are possible if certain conditions are fulfilled – as we will see later on.

4 The infinite multiplication constant k_∞ (F) and a numerical example

Before tackling Eq. (13) we first deal with the infinite multiplication constant k (F) being also a function of the neutron fluence. It is given by definition through

$$k_\infty(F) = \frac{\text{neutron-production}}{\text{neutron-absorption}} = \frac{\bar{\nu} \cdot \sigma^{fis}_f \cdot N_{fis}}{\sigma^{fis}_a \cdot N_{fis} + \sigma^p_a \cdot N_p + \sigma^{fp}_a \cdot N_{fp} + \Sigma^m_{a,0}} = \frac{\eta^{fis} \cdot e^{-F}}{e^{-F} + \alpha \cdot a \cdot e^{-\alpha F} + \frac{\beta}{\beta - 1} \cdot (e^{-F} - e^{-\beta F}) + b}$$

with

$$k_\infty(0) = \frac{\eta^{fis}}{1 + \alpha \cdot a + b} \quad (14)$$

being the classical expression for zero-power applications.

In order to be able to introduce quantitative figures we consider an infinite medium containing, for example, initially only three types of homogeneously mixed particles: graphite as the moderator, U-235 as the fissile fuel, and B-10 as poison.

The thermal nuclear data used are as follows:

(1) Graphite:

$$\rho = 1.6\ g/cm^3,\ \sigma^m_a = 4.8\ m\,b,\ \sigma^m_s = 4.8\ b,\ \sigma_{tr} = \sigma^m_s + \sigma^m_a \approx \sigma^m_s = 4.8\ m\,b,\ N_{m,0} = 8 \cdot 10^{22}/cm^3,\ \Sigma^m_{a,0} = 3.84 \cdot 10^{-4}/cm,$$

$$\Sigma^n_{tr} = 0.384/cm,\ D = {}^1/_3 \cdot \lambda_{tr} = 0.868\ cm.$$

(2) U-235:

$$\sigma^{fis}_f = 580\ b,\ \sigma^{fis}_c = 108\ b,\ \sigma^{fis}_a = \sigma^{fis}_f + \sigma^{fis}_c = 688\ b,$$

$$\eta^{fis} = 2.08$$

(3) Boron-10:

$$\sigma^p_a = 3\,837\ b,\ \alpha = \sigma^p_a/\sigma^{fis}_a = 5.577$$

(4) Fission product pairs: According to ref. [5], we use two groups of fission products, shown in Table 1, being in equilibrium with the actual burn-up dynamics. The total cross section of absorption is

$$\sigma^{fp}_a = 873\ b \text{ or } \beta = \sigma^{fp}_a/\sigma^{fis}_a = 1.27.$$

The Xe-poisoning effect is not contained. Since its time scale is measured in 10 h it has to be treated separately.

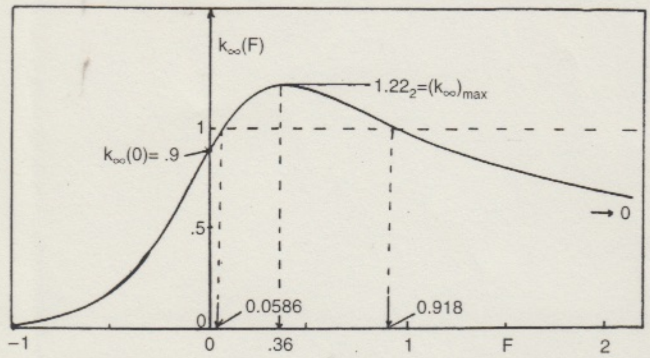

Fig. 1. The infinite multiplication constant k_∞ as function of the neutron fluence F. The continuation of the curve into the negative F-regime is only for academic interest. The increase of k_∞ due to the poison burn-up is clearly borne out because $dk_\infty/dF > 0$ for F = 0.

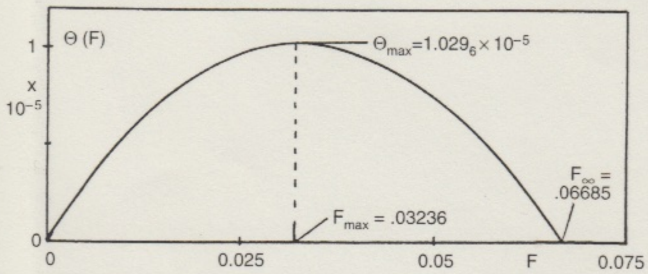

Fig. 2. Θ(F) from Eq. (13). There are two ground-states: $F_\infty = 0$ and $F_\infty = 0.06685$. Θ(F) can be expanded in a Fourier series where the fundamental mode is dominant (see text) since (F) is almost symmetrical around $F_\infty/2$.

If we assume that initially the infinite medium possesses $k_\infty(0) < 1$ we obtain from Eq. (14) the condition

$$N_{p,0} = \frac{1}{\alpha}\left[\frac{\eta^{fis}}{k_\infty(0)} - 1\right] \cdot N_{fis,0} - \Sigma^m_{a,0}/\sigma^p_a, \text{ in cm}^{-3} \quad (15)$$

For k (0) = 0.9, for example, we obtain – upon introducing the other numerical values – for the initial atomic particle density of the poison

$$N_{p,0} = .2351 \; N_{fis,0} - 1.001 \cdot 10^{17}, \text{ in cm}^{-3} \quad (16)$$

Taking $N_{fis,0} = 5 \cdot 10^{19}/cm^3$ we get $N_{p,0} = 1.1655 \cdot 10^{19}/cm^3$ and a = 0.2331 and b = $1.1163 \cdot 10^{-2}$.

Introducing these numbers, together with ß = 1.27 into Eq. (13) we obtain k_∞ (F) as shown in Fig. 1 exhibiting the fact that the burn-up of the poison increases k beyond unity.

5 Numerical solution of Eq. (13)

Introducing the above quantities into Eq. (13) together with $D\Sigma^{fis}_{a,0} = 2.986 \cdot 10^{-3}$ we derive

$$\begin{aligned} F'' + F' = \Theta(F) &= D \cdot \Sigma^{fis}_{a,0}\left[\left\{(\eta^{fis} - 1) + \frac{\beta}{\beta - 1}\right\}e^{-F}\right. \\ &\left. -a \cdot e^{-\alpha F} - \frac{1}{\beta - 1} \cdot e^{-\beta F} - \left\{(\eta^{fis} - 1) - a + 1\right\}\right] \\ &= 10^{-3} \cdot \left[17.27 \cdot e^{-F} - 0.696 \cdot e^{-5.577\,F}\right. \\ &\left. -11.059 \cdot e^{-1.27\,F} - 5.515\right] \end{aligned} \quad (17)$$

with the property that Θ(0) = 0. Fig. 2 displays the function Θ(F) as varying with the neutron fluence F.

Besides the first ground-state F = 0 we observe a second ground-state $F_\infty = 0.06685$ where also $\Theta(F_\infty) = 0$.

Since an analytical solution of Eq. (17) is not known we solved it numerically by using the SEQUENCE-generator of the TI-92 calculator and by applying a simple three point Euler algorithm for the derivatives. Fig. 3 visualizes the expected shock-like F function between the two mentioned asymptotic ground-states F = 0 and $F = F_\infty = 0.06685$. It is called the "kink" of the solution.

6 Approximations of Eq. (13)

Since F is almost symmetrical around $F_\infty/2$, when it is turned by 180°, we approximate Θ(F) in the range $0 \le F \le F_\infty$ by a Fourier series in the form

$$\Theta(F) = \sum_{n=1}^{\infty} A_n \cdot \sin\left(\frac{n \cdot \pi}{F_\infty}F\right) \quad (18)$$

with the coefficients being

$$A_n = \frac{2}{F_\infty}\int_0^{F_\infty} \Theta(F) \cdot \sin\left(\frac{n \cdot \pi}{F_\infty}F\right) \cdot dF. \quad (19)$$

The first coefficient, i. e. the amplitude of the fundamental mode A_1 and the amplitudes of the higher harmonics are given by

$$A_1 = 1.062 \cdot 10^{-5}, A_2 = 2.54 \cdot 10^{-7}, A_3 = 3.98 \cdot 10^{-7} \text{ etc.} \quad (20)$$

Since the amplitudes of the higher, harmonics are almost two orders of magnitude smaller than the amplitude of the fundamental mode we approximate Θ(F) by its fundamental mode (n = l) yielding a Sine-Gordon equation, possessing only an additional friction term on the LHS, given by

$$F'' + F' = A_1 \sin\left(\frac{\pi}{F_\infty}F\right) \quad (21)$$

with $F_\infty = 0.06685$ and $A_1 = 1.062 \cdot 10^{-5}$. As shown in the Appendix the friction term dominates the LHS of Eq. (21) because A_1 is very small. With F''« F' a further approximation of Eq. (21) is

$$F' = A_1 \cdot \sin\left(\frac{\pi}{F_\infty}F\right) \quad (22)$$

which can be integrated analytically by separating the variables. Because the integration variable is y = (x − vt)/D we derive for the neutron fluence soliton

$$F = \frac{2 \cdot F_\infty}{\pi} \cdot \arctan\left(e^{\frac{A_1 \pi}{F_\infty} \cdot y}\right) \quad (23)$$

For y → ∞ and y → − ∞ we obtain asymptotically the two ground-states F = 0 and $F = F_\infty$, as shown in Fig. 3. For our previous example, the suggested analytical approximation turns out to be in good agreement with the numerical solution. This property facilitates the discussions in the next steps.

The next step is to determine analytically the soliton shape of the neutron flux Φ. For this, we use Eq. (12) and Eq. (22). A peak neutron flux, Φ_0, can be introduced through

$$\begin{aligned} \Phi_0 &= -\frac{v}{D \cdot \sigma^{fis}_a}F'\left(F_{turn} = \frac{F_\infty}{2}\right) \\ &= -\frac{v \cdot A_1}{D \cdot \sigma^{fis}_a} \end{aligned} \quad (24)$$

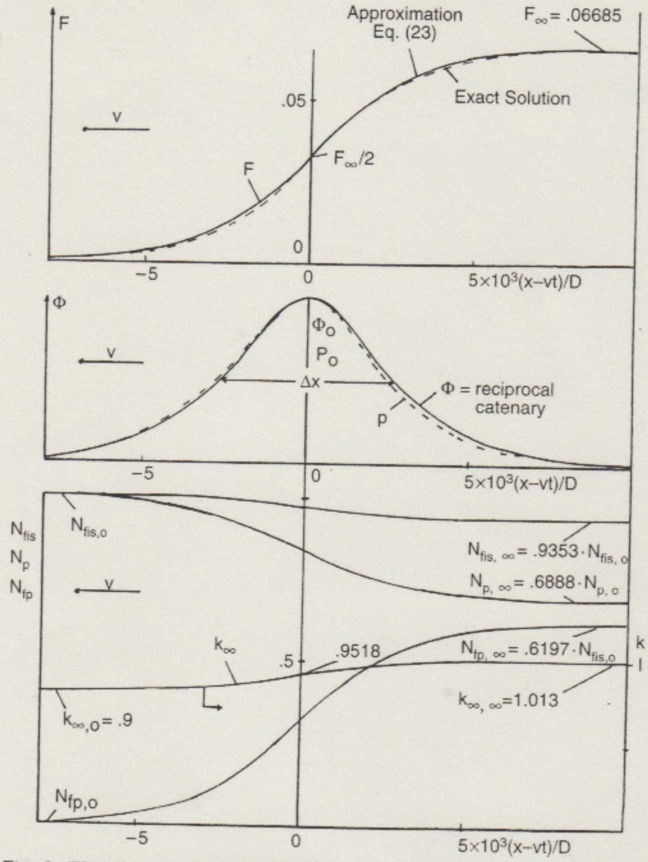

Fig. 3. The shapes of the neutron fluence, F, neutron flux, Φ, specific power density, p, solitons as well as the relevant atomic particle density solitons and k_∞ as function of the (dimensionless) variable (x-vt)/D. The approximation, Eq. (23), of F is a Gudermannian function, and Φ and p are approximately given by reciprocal catenaries or secans hyperbolicus functions. The velocity vector points to the left into the poisoned subcritical medium with $k_\infty(0) < 1$. All the shapes shown do not change during propagation (= isentropic transport).

Therefore, the velocity v of the solitons is

$$v = -\frac{D \cdot \sigma_a^{fis}}{A_1} \Phi_0 \tag{25}$$

and the flux soliton itself is given by

$$\Phi = \frac{\Phi_0}{\cosh\left[\frac{A_1 \cdot \pi}{F_\infty \cdot D}\left(x + \frac{D\,\sigma_a^{fis}}{A_1}\Phi_0 \cdot t\right)\right]} \tag{26}$$

which emphasizes the fact that the velocity is proportional to the amplitude – a typical non-linear effect in wave theory. The "plus sign" in the variable means that the wave is propagating to the left side as indicated in Fig. 3.

If, for instance, Φ_0 is assumed to be 10^{10} n/cm²s, the wave velocity turns out to be $|v| = 5.62 \cdot 10^{-7}$ cm/s = 17.7 cm/y. This value depends strongly on the initial subcriticality k_∞ (0).

The half-width, Δx, of the reciprocal catenary of Eq. (26) is

$$\Delta X = 0.8345 \frac{DF_\infty}{A_1} \approx 45.8\text{m} \tag{27}$$

which seems to be high for practical applications. However, this is only due to the example chosen. Nevertheless, it demonstrates the principle in an infinite medium.

The shape of the flux soliton ϕ [(x-vt)/D] is also shown in Fig. 3. The atomic particle densities N_{fis}, N_p, and N_{fp} are obtained by using Eqs. (3–5) and by inserting F from Eq. (23). Their variation as affected by the wave propagation is shown in Fig. 3, together with their asymptotic values before and behind the wave in its "wake" (see Appendix).

All these functions can be obtained in analytical and explicit form when introducing the above approximations. Not in all applications, however, these approximations are as good as in our actual example.

Furthermore, the power density, p, can be calculated using N_{fis} from Eq. (3), Φ from Eq. (23), and from Eq. (26) yielding

$$p[(x - vt)/D] = \varepsilon \cdot \sigma_f^{fis} \cdot N_{fis} \cdot \Phi \quad \text{in W/cm}^3$$

$$= \varepsilon \cdot \sigma_f^{fis} \cdot N_{fis,0} \cdot \Phi_0 \frac{\exp\left[-\frac{2F}{\pi} \cdot \arctan(e^y)\right]}{\cosh(y)} \tag{28}$$

with $y = [(A_1\ \pi)/(F_\infty\ D)] \cdot (x - vt)$ where $[(A_1\ \pi)/(F_\infty\ D)]$ is the wave number in cm^{-1} and ε = 200 MeV/fission = $3.2 \cdot 10^{-11}$ J/fission. For instance, if $\Phi_0 = 10^{12}$ n/cm²s we obtain $p_0 = 0.93$ W/cm³ = 930 kW/m³.

For the sake of completeness, Eq. (29) can also be expressed in terms of only circular and hyperbolic functions by

$$p = \varepsilon \cdot \Sigma_{f,0}^{fis} \cdot \Phi_0 \cdot \text{sech}(y) \cdot \text{cotg}\left[\frac{\text{gd}\{\frac{F_\infty}{\pi}\text{gd}(y)\}}{2}\right] \tag{29}$$

when the Gudermannian in the form

$$\text{gd}(z) = \int_{-\infty}^{z} \frac{dt}{\cosh(t)} = 2 \cdot \arctan(e^z) \tag{30}$$

is introduced being zero for $z \to -\infty$ and π for $z \to +\infty$ [6]. (By the way, this is the first application of the soliton theory in which nested Gudermannians play a role).

As shown in Fig. 3, p is slightly skew compared with Φ, mainly because N_{fis} is higher for $y < 0$ than for $y > 0$.

Since $F_\infty \ll \pi$, Eq. (29) can further be approximated by

$$p \approx \varepsilon \cdot \Sigma_{f,0}^{fis} \cdot \Phi_0 \cdot \text{sech}(y) \tag{31}$$

Hence, the shape of p becomes identical with that of the neutron flux Φ.

According to Eq. (32) the total power of the specific power density soliton in an infinite medium is given by

$$P = \int_{-\infty}^{+\infty} p \cdot d(x - vt)$$

$$= \varepsilon \Sigma_{f,0}^{fis} \cdot \Phi_0 \cdot \frac{F_\infty D}{A_1 \pi} \cdot \text{gd}(\infty)$$

$$= \varepsilon \Sigma_{f,0}^{fis} \cdot \Phi_0 \cdot F_\infty \cdot D/A_1 \quad \text{in W/cm}^2 \tag{32}$$

resulting for $\Phi = 10^{12}$ n/cm²s in P = 5.07 kW/cm².

7 Possible applications and conclusions

Of course, these ideas can also be applied to thermal light water reactor systems where the fuel possesses a fissile enrichment of about 3 % and where the burnable poison is Gd_2O_3.

In the opinion of the author, the ignition of a spatially propagating burn-up wave in a light water reactor will have sense only in small reactor cores like those of space-heating reactors. Their power levels are in the realm of 10–20 MW_{th} and their core lengths are in the range of 0.8–1.5 m [7]. The residence time of the fuel elements is approximately 11.5

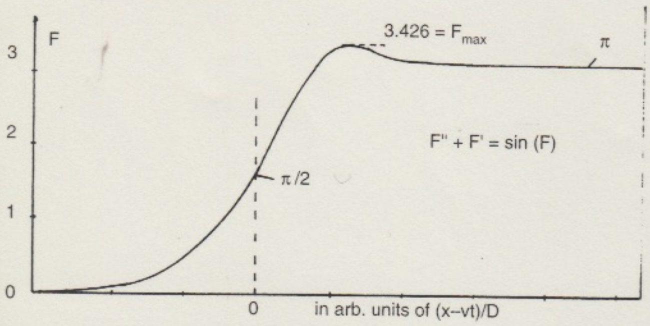

Fig. 4. The numerical solution of $F''+F' = sin(F)$ (in which the slope and curvature are equally weighted) as function of arbitrary units of (x-vt)/D. As can be seen, such a solution is physically not possible for our problem because a negative flux would result behind the maximum where $F' = 0$ and $F'' = sin(F_{max})$. In our problem F must increase monotonically as it is the case for the Gudermannian.

Table 1. Thermal absorption cross sections of fission products

	σ_a^{fp}, b	$T_{1/2}$ Isotope	$T_{1/2}$ Precursor	Fission yield, Y_i, %
Group I:				
Sm-149	$6.6 \cdot 10^4$	stable	50 h	1.150
Sm-151	$1.2 \cdot 10^4$	80 a	27 h	0.730
Cd-113	$2.5 \cdot 10^4$	stable	5.3 h	0.011
Eu-155	$1.4 \cdot 10^4$	stable	23 min	0.031
Group II:				
Eu-153	420	stable	47 h	0.15
Kr-83	205	stable	2.3 h	0.54
In-115	197	stable	54 h, 43 d	0.01
Nd-143	300	stable	13.8 h	5.90
	$\sigma_a^{fis} = \Sigma_{i=1}^{8} Y_i(\sigma_a^{fp})_i = 873\,b$			

years due to the relatively low load factor of 0.5 and due to the relatively low power density. Therefore, three core burn-ups correspond to the useful lifetime of the whole reactor system of 30–35 years.

In order to avoid any change or transports of fuel during this lifetime of the reactor, one could endow a 10 MW_{th} reactor at the beginning with a core of 2.4 m height instead of 0.8 m. In this core only the middle third would be critical at the beginning and would produce power. The upper and the lower third would be subcritical at the beginning due to a higher Gd_2O_3 poisoning. The parameters $k_\infty(0)$, F_∞, $N_{fis,0}$, $N_{p,0}$, etc. would be chosen in such a way that the two wave solitons, ignited by the power producing core in the middle and propagating in opposite directions, reach the two ends of the cylindrical core after about 30–35 years.

Then, the power producing phase is quenched automatically and the total burn-up is reached. Afterwards, all the internals including the reactor vessel with the irradiated fuel is removed and disposed of simultaneously in a nuclear waste repository. Thus, one initial fuel loading is adequate for its useful lifetime. We have called this the "battery concept".

The advantage is that fuel manipulations are not necessary during the lifetime of the reactor (except the initial loading and the unloading at the end). Compared with the possibility of making the whole length of the reactor critical from the beginning on, the soliton core design keeps the power producing volume smaller, i. e. more compact. The disadvantage is the higher initial fissile inventory, i. e. a more expensive core. It contains "self-shielded", stored fissile material which is needed only after a decade or so. This corresponds to "stored" capital and one has to pay interest rates for it leading to somewhat higher fuel cycle costs.

It is obvious that further research work is necessary, particularly with respect to the power control by means of reactivity control of the amplitude of the flux soliton. Also from the point of view of safety the soliton reactor will have preferable properties; a run away of the flux soliton is deterministically excluded.

Aside from these details, it should be noted that the primary idea of this work was to point out the possibility of a propagating critical core zone in thermal reactor systems.

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Appendix

A.1 Approximation of Eq. (21) by Eq. (22)

In order to show that the curvature (first term) can be neglected compared with the slope (second term) in Eq. (21)

$$F'' + F' = A_1 \cdot \sin\left(\frac{\pi}{F_\infty} F\right) \qquad (A.1)$$

we transform it into its standard form by the self-similar type of a formulation

$$F \rightarrow \frac{F_\infty}{\pi} F\left(\sqrt{\frac{A_1 \cdot \pi}{F_\infty}} \cdot y\right) \qquad (A.2)$$

resulting in

$$F'' + \sqrt{\frac{F_\infty}{A_1 \cdot \pi}} \cdot F' = \sin(F) \quad \text{or} \quad F'' + 44.8\, F' = \sin(F) \qquad (A.3)$$

if we insert $F_\infty = 0.06685$ and $A_1 = 1.062 \cdot 10^{-5}$. Since $A_1\pi/F_\infty$ is small compared with unity, the second (friction) term dominates the LHS of Eq. (A.4) and F'' can be neglected.

On the other hand, if $\sqrt{A_1\pi/F_\infty}$ was large compared with unity the same self-similar type of formulation in Eq. (A.2) leads again to Eq. (A.3). But then the first term (F'') would dominate the LHS and the second (friction) term could be neglected. If $\sqrt{A_1\pi/F_\infty}$ was in the order of magnitude of unity no approximation of Eq. (A.1) is possible.

Summarizing these procedures we can write Eq. (A.3) in the form

$$F'' + \kappa F' = \sin(F) \qquad (A.5)$$

If $\kappa \gg 1$ the approximation $F_1' = \sin(F_1)$ holds, possessing the solution $F_1 = 2\arctan(e^{y/\kappa})$. If $\kappa \ll 1$ the approximation $F_2'' = \sin(F_2)$ holds, possessing the solution $F_2 = 4\arctan(e^{y})$ if one remembers the multiple angle formula: $\sin(4\alpha) = 4\sin(\alpha)\cos(\alpha) - 8\sin^3(\alpha) - \cos(\alpha)$. The two ground-states of F_1 are 0 and π, and the two ground-states of F_2 are 0 and 2π.

Fig. 4 shows the numerical solution of $F'' + F' = \sin(F)$. It is obvious that such a solution is not possible in our physical application since a negative flux would result after the maximum. Therefore, we will always have a function as given by Eq. (A.5), which can be approximated accordingly.

A.2 Approximations of the atomic particle densities

Introducing the Gudermannian, F, of Eq. (23) into Eqs. (3–5) we can express the atomic particle densities in circular and hyperbolic functions. Using hyperbolic functions alone, the identity $gd(z) = 2 \arctan(e^z) = 2 \arctan(\tanh(z/2)) + \pi/2$ has to be introduced.

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